PHYSICAL REVIEW E VOLUME 58, NUMBER 2 AUGUST 1998

Breathing self-similar dynamics and oscillatory tails of the chirped dispersion-managed soliton

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(Received 17 March 1998)

An effective approach to describe a breathing soliton in systems with periodically varying dispersion is developed. A generalized solution of the propagation equation is presented in terms of chirped Gauss-Hermite orthogonal functions. As a particular example, developed theory describes both averaged slow evolution and rapid oscillations of the dispersion-managed soliton in fiber links. Self-similar structure of the main peak is described by a system of ordinary differential equations for root-mean-square width and integral chirp of the pulse. [S1063-651X(98)51108-7]

PACS number(s): 03.40.Kf, 42.81.Dp

An increasing demand for telecommunication services stimulates intensive research in the field of high-bit-rate optical data transmission. One of the key techniques to increase the capacity of fiber communication links is the so-called dispersion management [1–4]. Periodic dispersion management allows one to achieve stable optical signal transmission both in the linear and nonlinear regime. The recently discovered dispersion-managed (DM) soliton [5,6] is a new type of the information carrier with properties that differ substantially from that of a traditional fundamental soliton [soliton solution of the integrable nonlinear Schroedinger equation (NLSE)]. Main features of the DM soliton have already been well understood and described theoretically and by means of numerical modeling (see, e.g., [2–20]). However, due to a wide range of possible system configurations many practical and theoretical problems are still open. One such challenging problem is to describe structure of the oscillatory tails of the DM soliton. As has been shown in [13], the tails of the DM soliton are not self-similar and this changes the soliton profile during propagation along the compensation cell. In this Rapid Communication we develop a useful method to describe in a systematic way a self-similar dynamics of the core and non-self-similar oscillatory tails of the DM soliton. Using an orthogonal set of chirped Gauss-Hermite functions we derive average equation governing slow evolution and shape of the DM soliton. We have obtained a set of ordinary differential equations for the coefficients of expansion of DM soliton in terms of chirped Gauss-Hermite functions. We show that in the limit of a strong dispersion management (large variations of local dispersion) the self-similar soliton core and oscillatory tails can be well approximated by only two (the zero and fourth) terms in this expansion. The developed approach can be used advantageously in numerical modeling of the dynamics of arbitrary initial signal in the dispersion-managed communication systems.

Pulse evolution down the dispersion-managed optical transmission line is governed by a rather general model, the NLSE with periodic coefficients (see, e.g., [5–11] for details and notations):

$$iA_z+d(z)A_{tt}+c(z)|A|^2A=0. \tag{1}$$

Here in optical application the propagation distance z is normalized by the dispersion compensation period L normalized chromatic dispersion $d(z)=\tilde{d}(z)+\langle d\rangle$ represents the sum of a rapidly varying (over one compensation period) high local dispersion $\tilde{d}(z)$ and a constant residual dispersion $\langle d\rangle$ ($\langle d\rangle \ll \tilde{d}$); $c(z)$ accounts for power decay between amplifiers due to fiber loss. Angular brackets here and in the paper mean averaging over compensation period. Equation (1) possesses the conserved quantity $E=\int|A|^2dt$, that is, the energy of the system. Note that this equation describes many other physical applications, such as a stretched pulse generation in mode-locking fiber laser systems [16], propagation of high intensities beams in second-order nonlinear media with periodic poling, evolution of soliton in a periodically modulated nonlinear waveguide [19], and others. Therefore, we hope that the method developed in this paper can find applications in a range of similar physical problems.

It has already been shown in [15] that the dynamics of the DM soliton can be presented as self-similar evolution of the main peak accompanied by oscillations of far-field tails that have a non-self-similar structure [13]. Arbitrary input pulse propagating down the dispersion-managed line typically evolves into an asymptotic structure that presents a self-similar rapidly oscillating main peak and a dispersive pedestal [11]. By a proper choice of the parameters of the input pulse this radiation can be significantly suppressed. However, oscillatory far-field tails around the main peak cannot be entirely suppressed, because they present an unalienable part of the DM soliton. Below we discuss the origin of these oscillatory tails and present an effective method to describe far-field tails of the DM soliton. To describe rapid self-similar dynamics of the main peak let us consider the following [12] evolution of the integral-integral quantities [12] related to the pulse width, root mean square (RMS) width and the integral pulse chirp:

$$T_{int}(z)=\left[\frac{\int t^2|A|^2dt}{\int|A|^2dt}\right]^{1/2},\quad \frac{M_{int}(z)}{T_{int}(z)}=\frac{i}{4}\,\frac{\int t(AA_t^*-A^*A_t)dt}{\int t^2|A|^2dt}. \tag{2}$$

It is easy to check that the evolution of $T_{int}(z)$ and $M_{int}(z)$ is given by

$$\frac{dT_{int}}{dz}=4d(z)M_{int}(z),$$

$$\frac{d}{dz}(T_{int}M_{int})=\frac{4d(z)\int|A_t|^2dt-c(z)\int|A|^4dt}{4\int|A|^2dt}. \tag{3}$$

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To obtain a closed system of equations on T_{int} and M_{int} one has to express integrals $\int |A_t|^2 dt$ and $\int |A|^4 dt$ in terms of T_{int} and M_{int}. This is possible only under additional assumptions about the structure of the solution. Based on the results of numerical simulations [5,11] and the parabolic-law (for T_{int}) solution of the linear problem [10] let us make the following exact transformation of the function A [15]:

$$A(z,t)=\frac{Q[x,z]}{\sqrt{T(z)}}\exp\left(i\frac{M(z)}{T(z)}t^2\right), \quad \frac{t}{T(z)}. \tag{4}$$

Here, $T(z)$ and $M(z)$ are periodic functions of z to be defined below. We expect that the rapid oscillations of pulse width and chirp are accounted by $T(z)$ and $M(z)$ and slow evolution is given by $Q(x,z)$. Applying transformation (4) to Eq. (1) we obtain a partial differential equation for $Q(x,z)$,

$$i\frac{\partial Q}{\partial z}+\frac{d}{T^2}Q_{xx}+\frac{c(z)}{T}|Q|^2Q-[TM_z-MT_z+4d(z)M^2]x^2Q$$
$$=i\frac{T_z-4d(z)M}{2T}Q+i\frac{T_z-4d(z)M}{T}xO_x. \tag{5}$$

Integral pulse characteristics T_{int} and M_{int} are expressed in terms of T and M and integrals of $Q(x,z)$, $Q^*(x,z)$ (and their derivatives),

$$T_{int}(z)=T(z)\left[\frac{\int x^2|Q(x,z)|^2dx}{\int |Q(x,z)|^2dx}\right]^{1/2},$$

$$\frac{M_{int}(z)}{T_{int}(z)}=\frac{M(z)}{T(z)}+\frac{i}{4T(z)^2}\frac{\int x(QQ_x^*-Q^*Q_x)dx}{\int x^2|Q|^2dx}. \tag{6}$$

Straightforward calculations yield equations for integral characteristics of the field Q,

$$\frac{d}{dz}\langle x^2\rangle=\frac{2d(z)}{T^2}W(z)-2\frac{T_z-4d(z)M}{T}\langle x^2\rangle,$$

$$\langle x^2\rangle=\frac{\int x^2|Q|^2dx}{\int |Q|^2dx}, \quad W(z)=\frac{\int x(QQ_x^*-Q^*Q_x)dx}{\int |Q|^2dx}, \tag{7}$$

and

$$\frac{d}{dz}W(z)=-4\langle x^2\rangle T\left(\frac{dM}{dz}-\frac{d(z)C_1}{T^3}-\frac{c(z)C_2}{T^2}\right)$$
$$+4M(T_z-4d(z)M)\langle x^2\rangle,$$

$$C_1=\frac{\int |Q_x|^2dx}{\int x^2|Q|^2dx}, \quad C_2=\frac{\int |Q|^4dx}{4\int x^2|Q|^2dx}. \tag{8}$$

Note that up to now this is the *exact* transform from A to Q. As a next step, we assume now that the DM soliton in the leading order is close to self-similar structure given by Eq. (4) under additional constraint $\partial \arg Q/\partial x=0$. In other words, we assume that first, in the leading order the phase factor Mt^2/T in the transformation (4) describes the pulse chirp in the energy-containing central part (pulse core) and second, RMS width for the transformed field Q does not vary with z: $d\langle x^2\rangle/dz=0$. These two assumptions lead to condition $W(z)=0$ and to the equations on the local functions T and M,

$$\frac{dT}{dz}=4d(z)M. \tag{9}$$

Further, it can be easily checked that under condition $\partial/\partial x$ (arg Q)=0 quantities C_1 and C_2 don't depend on z. As a result, we get a second required ordinary differential equation,

$$\frac{dM}{dz}=\frac{d(z)C_1}{T^3}-\frac{c(z)C_2}{T^2}. \tag{10}$$

Equations (9) and (10) were first introduced in the context of DM cascaded systems by Gabitov and Turitsyn in [10] using the variational approach. We emphasize that here these equations are derived directly from basic Eq. (1) under a few reasonable assumptions justified by numerical simulations. Below we present a rigorous way to account for small deviations of the DM pulse from a self-similar wave form assumed above. Note also that the pulse shape is not fixed in this approach. Pulse power, width, and form affect rapid dynamics only through integral coefficients C_1 and C_2. Additionally, one can move C_1 to the parameter in the third term in Eq. (10) by transform $T_{old}=C_1^{1/4}T_{new}$ and $M_{old}=C_1^{1/4}M_{new}$ that keeps the structure of Eqs. (9) and (10) the same with new $C_{1new}=1$ and $C_{2new}=C_2C_1^{-3/4}$ and the corresponding change of the boundary conditions. Making use of this observation in what follows we slightly change notation using Eqs. (9) and (10) with $C_1=1$ and introducing $N^2=C_2C_1^{-3/4}$. It is interesting to note that a similar set of simple equations can be derived for the pulse characteristics in the spectral domain [8]. In [8] equations in the spectral domain have been introduced by means of the variational approach. We derive now the basic ordinary differential equation (ODE) model in the spectral domain using RMS pulse characteristics similar to those considered above. Let us introduce the RMS pulse spectral bandwidth and chirp as

$$\Omega_{RMS}(z)=\left[\frac{\int \omega^2|A|^2d\omega}{\int |A|^2d\omega}\right]^{1/2},$$

$$Y_{RMS}(z)=\frac{i}{4}\frac{\int \omega(A^*\partial A/\partial\omega-A\partial A^*/\partial\omega)d\omega}{\int \omega^2|A|^2d\omega}=\frac{T_{int}M_{int}}{\Omega_{RMS}^2}. \tag{11}$$

Evaluating first derivatives of these quantities with z and assuming as above the self-similar structure of the core of a pulse we get two equations for the Ω_{RMS} and Y_{RMS},

$$\frac{d\Omega_{RMS}}{dz}=-\frac{4N^2c(z)C_2Y_{RMS}\Omega_{RMS}^4}{\langle x^2\rangle[C_1+4Y_{RMS}^2\Omega_{RMS}^4/\langle x^2\rangle]^{3/2}}, \tag{12}$$

$$\frac{dY_{RMS}}{dz}-\tilde{d}(z)$$
$$=\langle d\rangle-\frac{N^2c(z)C_2[C_1\langle x^2\rangle-4Y_{RMS}^2\Omega_{RMS}^4]}{\langle x^2\rangle\Omega_{RMS}[C_1+4Y_{RMS}^2\Omega_{RMS}^4/\langle x^2\rangle]^{3/2}}. \tag{13}$$

Here C_1, C_2, $\langle x^2\rangle$ are integrals defined above. Advantage of these equations is that after trivial redefinition of the function $Y_{RMS}(z)=Y_{RMS}^{(new)}(z)+\tilde{R}_0(z)$ [with known function $\tilde{R}_0(z)$

found from $d\tilde{R}_0/dz=\tilde{d}(z)$] and treating nonlinearity and residual dispersion as small perturbations during one period we obtain ODEs with small parameters on the right-hand-side [8,17,18]. Using this approach the conditions of the stationary propagation of the DM soliton in the system with strong dispersion management have been obtained in [17,18]. For the Gaussian trial function $A(t,z)=NB_0\exp[iM(z)t^2/T(z)]\exp\{-t^2/[2T^2(z)]+i\lambda(z)\}/\sqrt{T(z)\sqrt{\pi}}$ such conditions give $|B_0|^2=2\sqrt{2\pi}$, $\langle dM\rangle=0$, $\langle d(1/T^2+4M^2)\rangle=N^2\langle c/T\rangle$ [17,18]. Below these conditions of the periodic DM pulse propagation will be obtained by exact expansion of the DM soliton in the complete set of chirped Gauss-Hermite functions.

Now we consider slow evolution of the DM soliton and will describe the dynamics of non-self-similar oscillatory tails. To remove from Eq. (1) rapid self-similar dynamics that occurs due to large variations of the local dispersion let us apply the following [15] the exact transformation:

$$A(t,z)=N\exp\left[i\frac{M(z)}{T(z)}t^2\right]\frac{Q(x,z)}{\sqrt{T(z)}},\quad x=\frac{t}{T(z)}. \tag{14}$$

Here we also slightly change notation (compared with the above consideration), putting power coefficient N explicitly in the field transform. T and M in Eq. (14) are periodic solutions of the equations

$$\frac{dT}{dz}=4d(z)M,\quad \frac{dM}{dz}=\frac{d(z)}{T^3}-\frac{c(z)N^2}{T^2}. \tag{15}$$

Here N is a constant that is determined by requirement that T and M are periodic solutions to Eq. (15). We obtain then a partial differential equation for $Q(x,z)$,

$$i\frac{\partial Q}{\partial z}+\frac{d}{T^2}(Q_{xx}-x^2Q)+\beta(z)(|Q|^2Q+x^2Q)=0,$$

$$\beta(z)=\frac{c(z)N^2}{T}. \tag{16}$$

Since the system of the eigenfunctions of the harmonic oscillator is complete, we can expand $Q(x,z)$ using the orthogonal normalized Gauss-Hermite functions $Q(x,z)=\Sigma_n B_n(z)f_n(x)\exp[iR(z)\lambda_n]$ with

$$(f_n)_{xx}-x^2f_n=\lambda_n f_n,\quad \lambda_n=-1-2n,$$

$$f_n(x)=\frac{1}{\sqrt{2^n n!\sqrt{\pi}}}\exp\left(-\frac{x^2}{2}\right)H_n(x). \tag{17}$$

Here, $H_n(x)$ is the nth-order Hermite polynomial and coefficients B_n are given by the ordinary scalar product in $\mathcal{L}^2$ with f_m.

Function $R(z)$ is found from $dR/dz=d(z)/T(z)^2-\langle d/T^2\rangle$. Inserting this expansion into Eq. (16), after scalar multiplication with f_m we obtain a system of ordinary differential equations for the coefficients B_m,

$$i\frac{dB_m}{dz}+\left\langle\frac{d}{T^2}\right\rangle\lambda_m B_m+\beta(z)\sum_{n=0}^{\infty}e^{i(\lambda_n-\lambda_m)R(z)}S_{n,m}B_n$$
$$+\beta(z)\sum_{n,l,k}e^{i(\lambda_n+\lambda_l-\lambda_k-\lambda_m)R(z)}B_nB_lB_k^*V_{m,n,l,k}=0. \tag{18}$$

Here we introduce notation $S_{n,m}=\int_{-\infty}^{+\infty}f_m(x)x^2f_n(x)dx$, $V_{n,m,l,k}=\int_{-\infty}^{+\infty}f_n(x)f_m(x)f_l(x)f_k(x)dx$. Since integrals of the form $\int x^ne^{-\alpha x^2}$ can be calculated analytically, it is possible to determine any $S_{n,m}$ and $V_{n,m,l,k}$. For instance, $S_{0,0}=0.5$, $S_{0,2}=1/\sqrt{2}$, $S_{2,2}=5/2$, $S_{4,4}=9/2$ and first few coefficients $V_{n,m,l,k}$ are

$$V_{0,0,0,0}=\frac{1}{\sqrt{2\pi}},\quad V_{1,1,0,0}=\frac{1}{2}\frac{1}{\sqrt{2\pi}},\quad V_{2,0,0,0}=-\frac{1}{4}\frac{1}{\sqrt{\pi}}.$$

Equation (18) can be averaged directly [in contrast to the master Eq. (1)]. Averaging over one period gives

$$i\frac{dU_m}{dz}+\left\langle\frac{d}{T^2}\right\rangle\lambda_mU_m+\sum_n\langle\beta(z)e^{i(\lambda_n-\lambda_m)R(z)}\rangle S_{n,m}U_n$$
$$+\sum_{n,l,k}\langle\beta(z)e^{i(\lambda_n+\lambda_l-\lambda_k-\lambda_m)R(z)}\rangle U_nU_lU_k^*V_{m,n,l,k}=0. \tag{19}$$

Here $B_m=U_m+\eta_m+\cdots$ is composed as a sum of slowly (U_m) and rapidly (η_m) varying parts ($d\eta_m/dz\gg\eta_m$) and $\eta_m\ll U_m$. The stationary solution of Eq. (19) having the form $U_m=F_m\exp(ikz)$ with F_m nondependent on z presents the expansion of the DM soliton in terms of chirped Gauss-Hermite functions for given dispersion map. The path-averaged equation (19) allows one to describe in a rigorous way properties of DM solitons and more generally propagation of any input signal for arbitrary dispersion map. Rapid convergence, which takes place for a bell-shaped pulse, means that the localized pulse will be well presented by a limited number of terms in the expansion. This makes such a basis very useful in different practical applications. As a particular example, consider now DM soliton close to Gaussian in the core assuming $F_0\gg F_m$ for $m\neq 0$. First let us keep only two first modes. Because of the symmetry it will be $m=0$ and $m=2$. We assume that $F_0\gg F_2$ and neglect as a first step all other modes and terms quadratic and qubic in F_2.

$$-kF_0+\left\langle\frac{d}{T^2}\right\rangle\lambda_0F_0+\langle\beta\rangle S_{0,0}F_0+\langle\beta(z)e^{-4iR(z)}\rangle S_{0,2}F_2$$
$$+\langle\beta\rangle|F_0|^2F_0V_{0,0,0,0}+V_{0,0,0,2}(\langle\beta e^{-4iR(z)}\rangle 2|F_0|^2F_2$$
$$+\langle\beta e^{4iR(z)}\rangle|F_0|^2F_2^*)=0, \tag{20}$$

$$-kF_2+\left\langle\frac{d}{T^2}\right\rangle\lambda_2F_2+\langle\beta\rangle S_{2,2}F_2+\langle\beta(z)e^{4iR(z)}\rangle S_{2,0}F_0$$
$$+\langle\beta e^{4iR(z)}\rangle|F_0|^2F_0V_{0,0,0,2}+V_{2,0,0,2}(\langle\beta\rangle 2|F_0|^2F_2$$
$$+\langle\beta e^{8iR(z)}\rangle|F_0|^2F_2^*)=0. \tag{21}$$

It is interesting that these equations always have an exact solution that in the leading order approximates DM soliton in this (strong management) limit. Namely,

$$|F_0|^2=-S_{2,0}/V_{0,0,0,2}=2\sqrt{2\pi},\quad F_2=0,$$

$$k=-\langle d/T^2\rangle+\langle\beta\rangle\left(\frac{1}{2}+\frac{|F_0|^2}{\sqrt{2\pi}}\right)=-\langle d/T^2\rangle+2.5\langle\beta\rangle. \tag{22}$$

Note that this solution gives *exactly* the same conditions of the stationary propagation of DM pulse obtained above by the RMS momentum method. This can be considered as a solution that gives the best approximation of the DM soliton by a Gaussian-shaped pulse. In a true DM soliton B_2 is not exactly zero due to higher order terms neglected in the above consideration, but as it has been observed in [21] B_2 is indeed smaller than the next few terms in the expansion. More involved calculations allow one to find similar solution considering a three-mode approximation for $m=0,2,4$. Again, requiring that $B_2=0$ and neglecting terms quadratic and qubic in F_4 we can solve the three-mode model and obtain

$$F_4=F_0|F_0|^2\frac{J_{04}^*J_{44}F_0^2-J_{04}(K_0+K_2|F_0|^2)}{(K_0+K_2|F_0|^2)^2-|J_{44}|^2|F_0|^4},$$

$$J_{04}=V_{0,0,0,4}\langle\beta e^{8iR}\rangle,\quad J_{44}=V_{4,0,0,4}\langle\beta e^{16iR}\rangle,$$

$$K_0=\langle d/T^2\rangle(\lambda_4-\lambda_0)+\langle\beta\rangle(S_{4,4}-S_{0,0}),$$

$$K_2=\langle\beta\rangle(2V_{4,0,0,4}-V_{0,0,0,0}). \tag{23}$$

Here F_0^2 (F_0 is real without loss of generality) is found from the cubic algebraic equation

$$\langle\beta e^{4iR}\rangle(S_{2,0}+V_{0,0,0,2}F_0^2)[(K_0+K_2F_0^2)^2-|J_{44}|^2F_0^4]$$
$$+\langle\beta e^{-4iR}\rangle(S_{2,4}+2V_{2,0,0,4}F_0^2)F_0^2[J_{04}^*J_{44}F_0^2-J_{04}$$
$$\times(K_0+K_2F_0^2)]+\langle\beta e^{12iR}\rangle V_{2,0,0,4}F_0^4[J_{04}J_{44}^*F_0^2-J_{04}^*$$
$$\times(K_0+K_2F_0^2)]=0. \tag{24}$$

By construction we are interested in the root close to $F_0^2=-S_{2,0}/V_{0,0,0,2}=2\sqrt{2\pi}$. Note that in the proper expansion of the sech-shaped pulse in the basis of chirped Gauss-Hermite functions the terms with $m=4k+2$, $k=0,1,2,\cdots$ are zero [20]. This indicates that the developed approximation based on the zero and fourth modes can effectively allow one to describe with good accuracy the evolution of the DM soliton form with increasing of the map strength. Details will be presented elsewhere. Neglecting the nonlinear terms for $m\geqslant 2$ in the expansion the coefficient F_0 can also be expressed through the soliton energy $F_0\approx\sqrt{E/N}$. The power distribution is then given by $[F_n=|F_n|\exp(\Phi_n)]$

$$|A(z,t)|^2=\frac{E}{T(z)}\frac{\exp[-t^2/T^2(z)]}{\sqrt{\pi}}+\frac{2N\sqrt{E}}{T(z)}\sum_{n=1}^{n=\infty}f_0f_{2n}|F_{2n}|\cos[4nR(z)-\Phi_{2n}]. \tag{25}$$

The last term here is responsible for non-self-similar oscillations of the tails inside compensation cell. Note that non-Gaussian oscillating tails are always present in the DM soliton, even if the main peak is very close to the Gaussian shape.

In conclusion, we have presented a direct method to describe both self-similar core and the oscillating tails of a pulse propagating in dispersion-managed systems. Using a complete set of chirped Gauss-Hermite functions we describe the evolution of an arbitrary DM pulse and present a simple two-mode model describing the DM soliton in systems with strong dispersion management.

The support of RFBR (Grant No. 96-02-19131-a) is acknowledged.

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